Differential equations system

Solving the differential equations system

```
Clear["Global`*"];
```

```
ClearAll[x, y]
```

We have the system:

x'[t]-2y[t]=0

y'[t]+x[t]+y[t]=0

with the initial conditions: x[0]=1; y[0]=2

```
sol = DSolve[{D[x[t], t] == 2 * y[t], D[y[t], t] == -x[t] - y[t], x[0] == 1,
   y[0] == 2}, {x[t], y[t]}, t]
```

$$\left\{\left\{x[t] \to \frac{1}{7} e^{-t/2} \left(7 \operatorname{Cos}\left[\frac{\sqrt{7}\, t}{2}\right] + 9\sqrt{7} \operatorname{Sin}\left[\frac{\sqrt{7}\, t}{2}\right]\right),\right.\right.$$

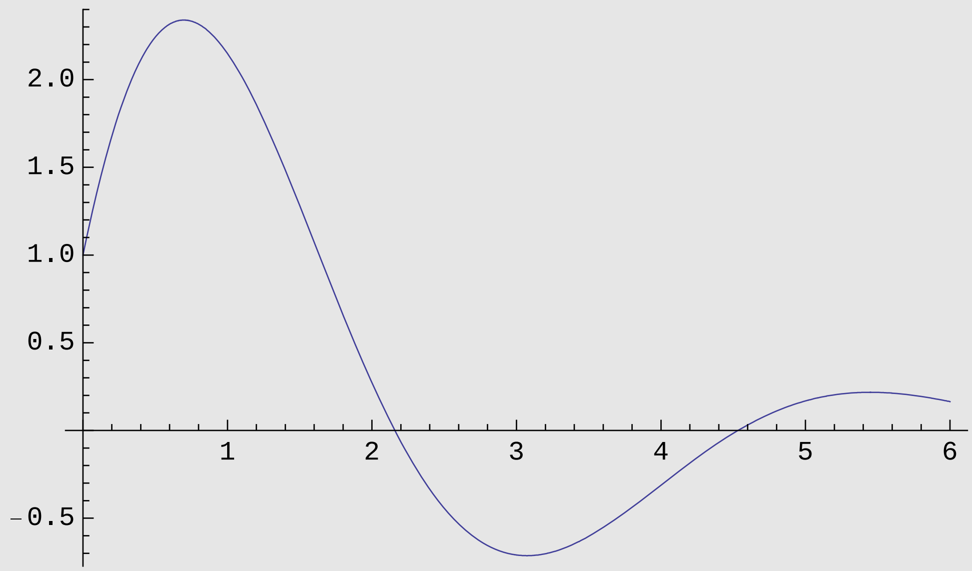
$$\left.\left. y[t] \to -\frac{2}{7} e^{-t/2} \left(-7 \operatorname{Cos}\left[\frac{\sqrt{7}\, t}{2}\right] + 2\sqrt{7} \operatorname{Sin}\left[\frac{\sqrt{7}\, t}{2}\right]\right)\right\}\right\}$$

```
sol1[t_] = First[x[t] /. sol]
sol2[t_] = First[y[t] /. sol]
```

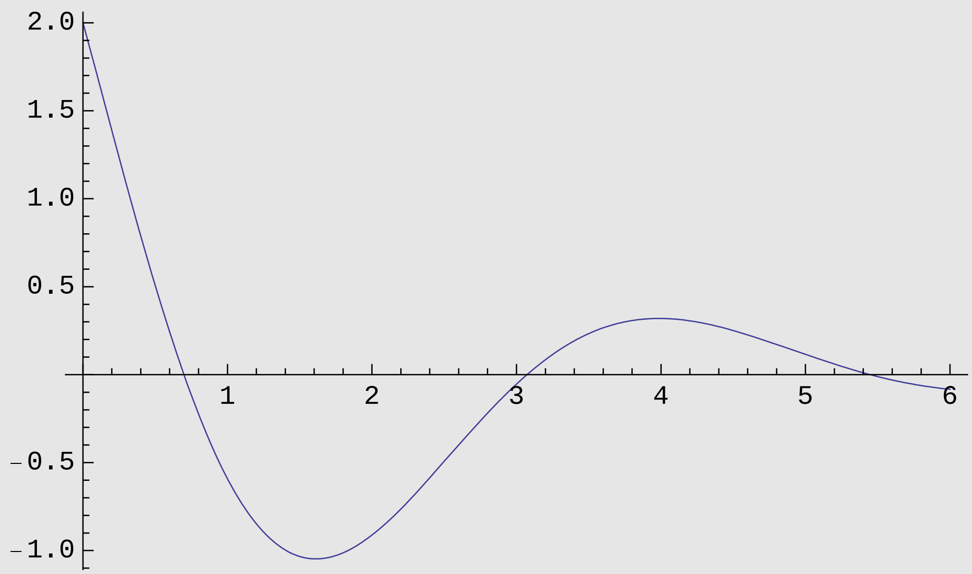
$$\frac{1}{7} e^{-t/2} \left(7 \operatorname{Cos}\left[\frac{\sqrt{7}\, t}{2}\right] + 9\sqrt{7} \operatorname{Sin}\left[\frac{\sqrt{7}\, t}{2}\right]\right)$$

$$-\frac{2}{7} e^{-t/2} \left(-7 \operatorname{Cos}\left[\frac{\sqrt{7}\, t}{2}\right] + 2\sqrt{7} \operatorname{Sin}\left[\frac{\sqrt{7}\, t}{2}\right]\right)$$

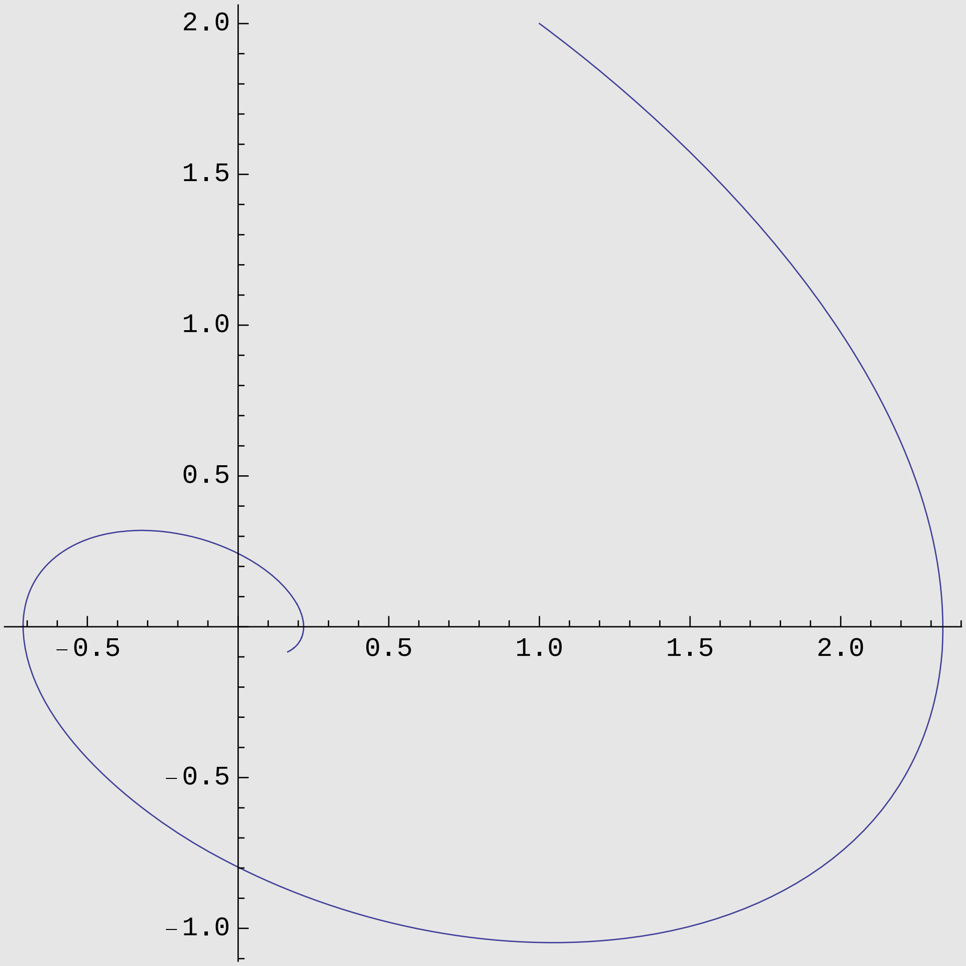
```
Plot[sol1[t], {t, 0, 6}]
```




```
Plot[sol2[t], {t, 0, 6}]
```



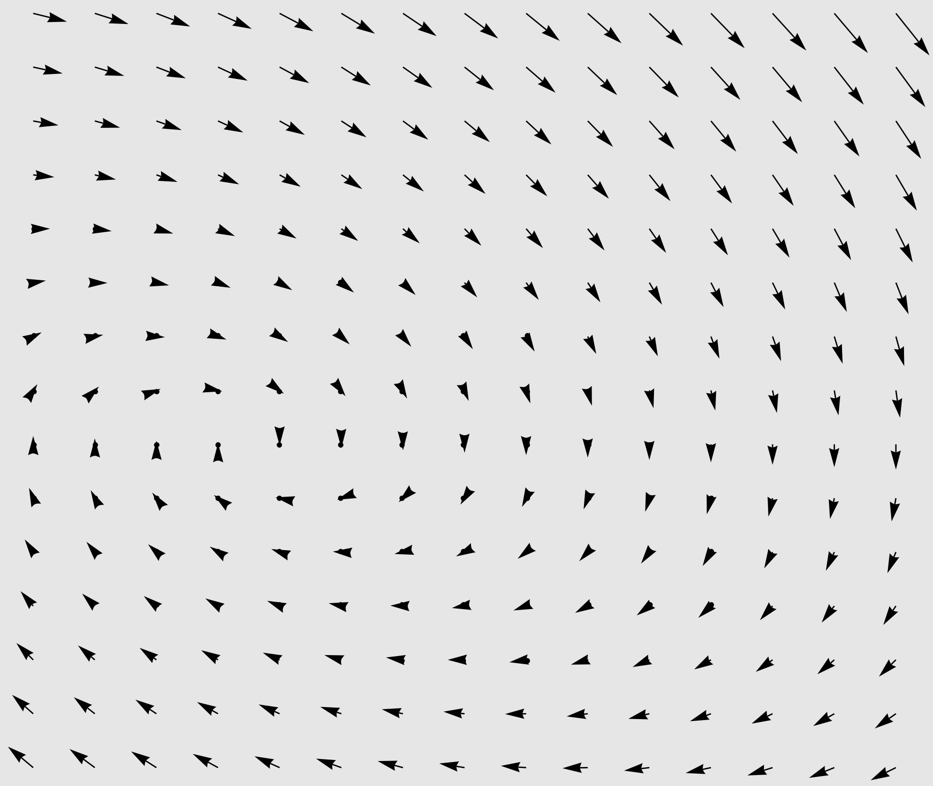
```
para1 = ParametricPlot[{sol1[t], sol2[t]}, {t, 0, 6}]
```



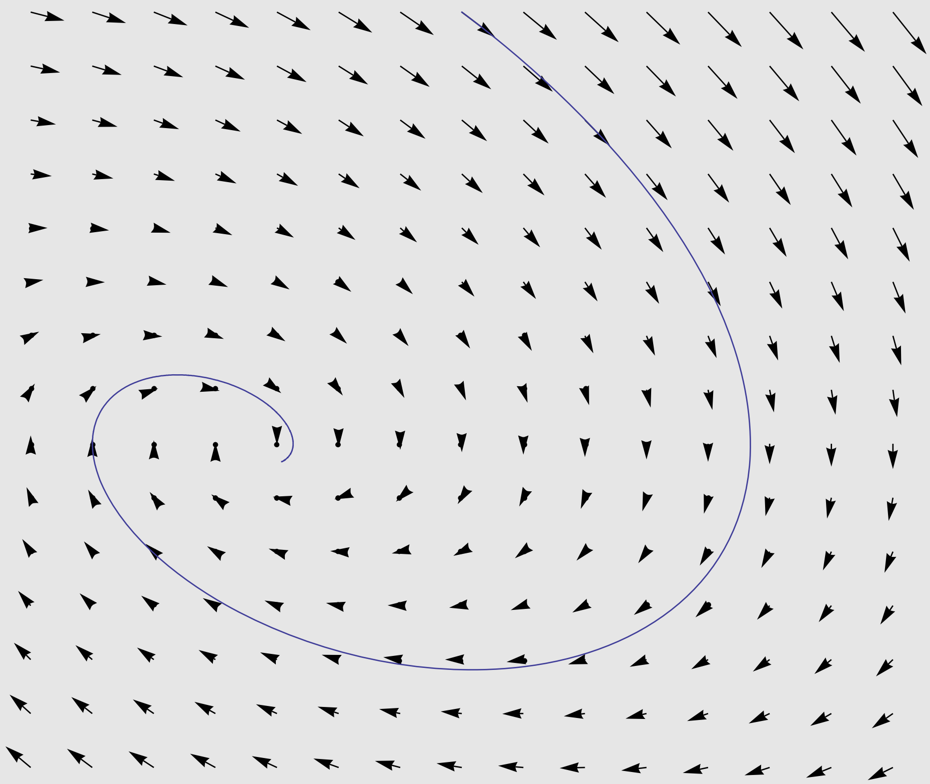

```
Needs["VectorFieldPlots`"]
```

General::obspkg :
VectorFieldPlots` is now obsolete. The legacy version being loaded may conflict with current *Mathematica* functionality. See the Compatibility Guide for updating information. ≫

```
vectorplt = (Needs["VectorFieldPlots`"];
  VectorFieldPlots`VectorFieldPlot[{2 y, -x - y}, {x, -1, 3},
   {y, -1.5`, 2}])
```



```
Show[vectorplt, para1]
```



We have the system:

x'[t]+x[t]-2y[t]-1=0

y'[t]-x[t]-3z[t]-1=0

z'[t]-2y[t]+z[t]=0

with the intial conditions: x[0]=6; y[0]=2; z[0]=4

```
soln =
 Simplify[
  DSolve[{D[x[t], t] == -x[t] + 2 * y[t] + t,
    D[y[t], t] == x[t] + 3 * y[t] - z[t] + 1, D[z[t], t] == 2 * y[t] - z[t],
    x[0] == 6, y[0] == 2, z[0] == 4}, {x[t], y[t], z[t]}, t]]
```

$$\left\{\left\{x[t] \to \frac{1}{72} e^{-t} \left(369 + 103 e^{4t} - 108 t + 8 e^{t} (-5 + 3 t)\right), y[t] \to \frac{1}{36} \left(-4 - 27 e^{-t} + 103 e^{3t} - 12 t\right), z[t] \to \frac{1}{72} e^{-t} \left(153 + 103 e^{4t} + e^{t} (32 - 48 t) - 108 t\right)\right\}\right\}$$

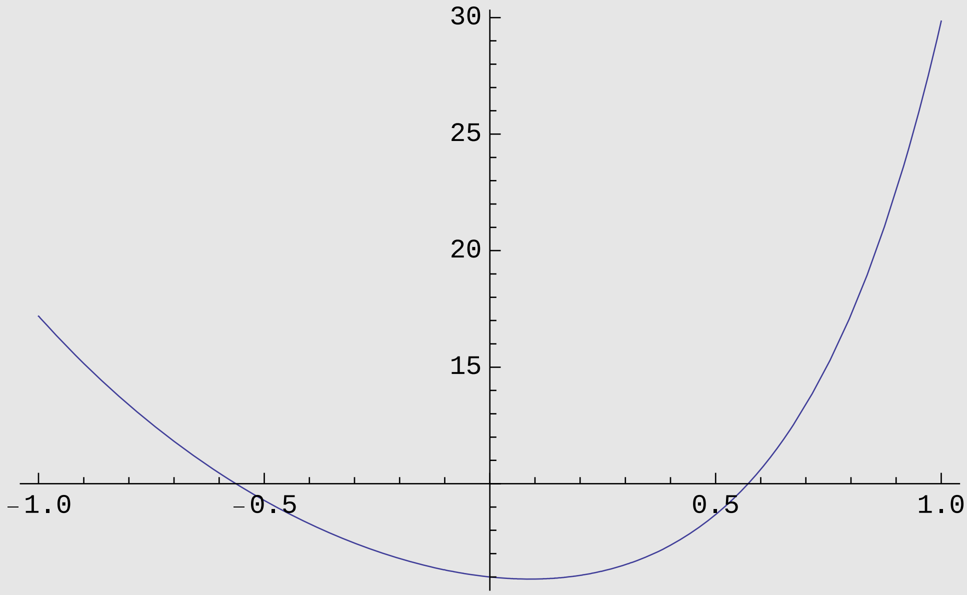
```
sol1[t_] = First[x[t] /. soln]
sol2[t_] = First[y[t] /. soln]
sol3[t_] = First[z[t] /. soln]
```

$$\frac{1}{72} e^{-t} \left(369 + 103 e^{4t} - 108 t + 8 e^{t} (-5 + 3 t)\right)$$

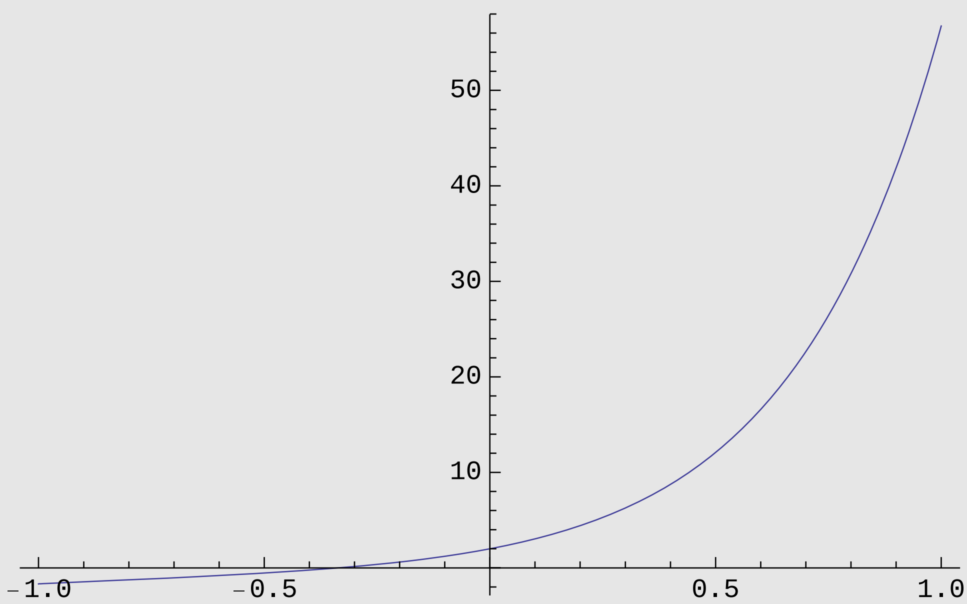
$$\frac{1}{36} \left(-4 - 27 e^{-t} + 103 e^{3t} - 12 t\right)$$

$$\frac{1}{72} e^{-t} \left(153 + 103 e^{4t} + e^{t} (32 - 48 t) - 108 t\right)$$

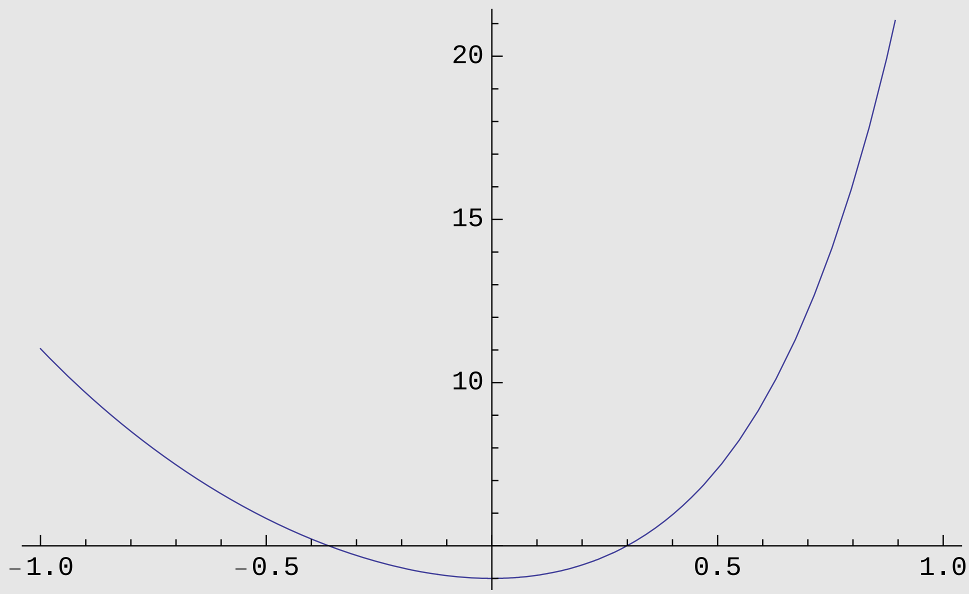
```
Plot[sol1[t], {t, -1, 1}]
```



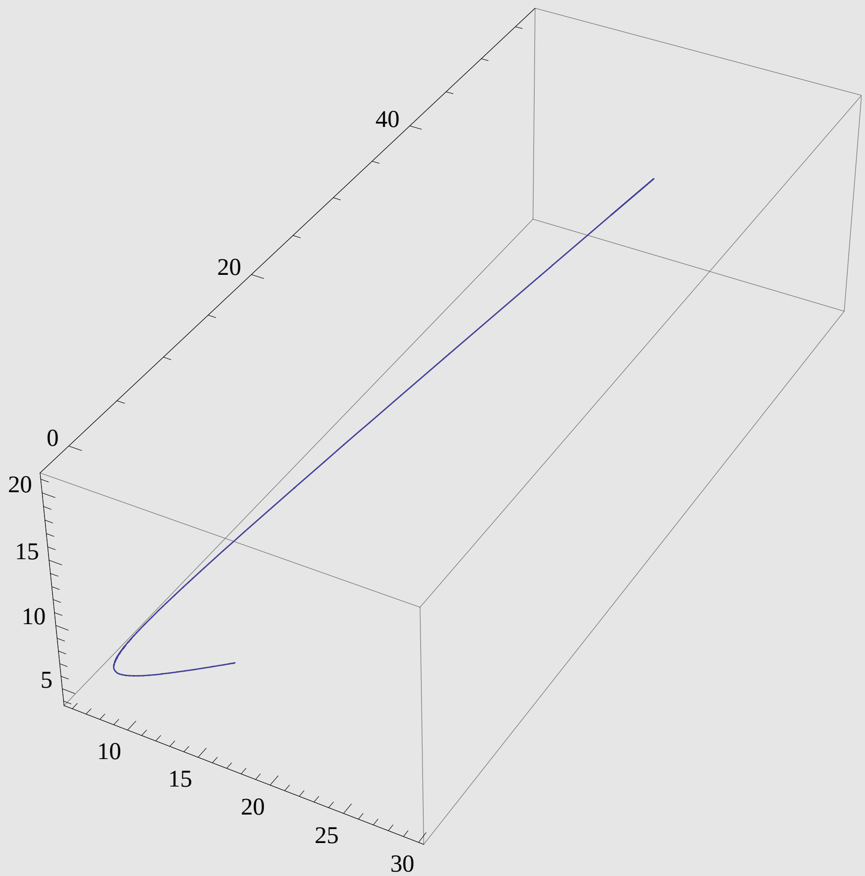

```
Plot[sol2[t], {t, -1, 1}]
```



```
Plot[sol3[t], {t, -1, 1}]
```



```
plot3d = ParametricPlot3D[{sol1[t], sol2[t], sol3[t]}, {t, -1, 1}]
```

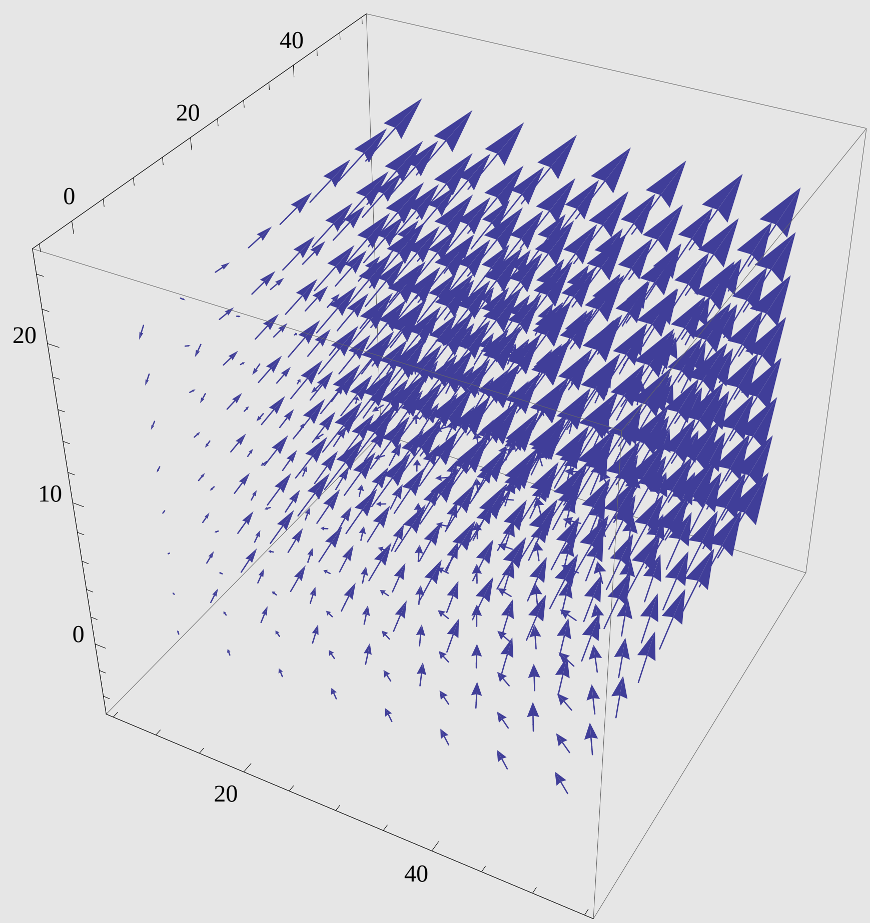



```
Needs["VectorFieldPlots`"]
```

```
t = 0
```

```
0
```

```
vplt3d = (Needs["VectorFieldPlots`"];
  VectorPlot3D[{-x + 2 y + t, x + 3 y - z + 1, 2 y - z}, {x, 10, 50},
   {y, 0, 50}, {z, 0, 20}])
```



```
Show[plot3d, vplt3d]
```